

Gemma in April: A Matrix-like Parallel Programming Architecture on OpenCL

Abstract—Nowadays, Graphics processing units (GPUs), as a kind of massive parallel processor, have been widely used in general computing tasks. Although there have already been mature development tools, it is not a trivial task for programmers to write high-performance GPU programs. Based on this consideration, we design a novel parallel computing architecture. The architecture includes a general programming model, named Gemma, and a programming framework, named April. Gemma is based on generalized matrix operations, and helps to alleviate the difficulty of describing parallel algorithms. Based on Gemma and OpenCL, April can automatically execute Gemma described tasks with OpenCL. In particular, April can automatically 1) choose the best parallel algorithm and mapping scheme, and generate OpenCL kernels, 2) control the running of OpenCL kernels as well as data transfer. Our experimental results show that the code length of April based implementation is far less than that of the direct OpenCL based implementation, and the performance of April-based application is competitive.

I. INTRODUCTION

In the battle to achieve higher computing performance, Graphics Processing Unit (GPU) emerges as a new and promising type of processors for parallel computing. Because of the inherent parallelism of graphics rendering algorithms, which deal with a large number of independent pixels, the GPU hardware architecture is designed for parallel computing. Besides, GPUs are designed to floating point operations. These features make modern GPU different from the mainstream general purpose processors in both structure and performance.

The architecture of GPU is suitable for not only graphics rendering algorithms, but also general parallel algorithms in a wide variety of application domains. Besides, today, a high-end GPU has orders of magnitude higher computation ability and memory bandwidth than a high-end CPU. For the reasons above, more and more general-purpose computation applications get mapped to graphics hardware.

However, as result of its highly specialized parallel architecture, we encounter parallel programming problems when computing on GPUs, such as parallel task partitioning, communication between threads, synchronization, load balancing, optimization of memory access mode and so on. Hence, designing an efficient GPU program is not easy, which requires the programmer's awareness of the detailed architecture of the underlying chip, hence extends the development cycle and raises the threshold of GPU parallel computing.

To lower the difficulty of writing parallel programs on GPU and maximize the application efficiency, we design a novel parallel computing architecture, which can help to alleviate the difficulty of General-Purpose Computing on GPUs. The architecture includes a general programming model, named Gemma, and a programming framework, named April. Gemma is based on generalized matrix operations, and helps to alleviate the difficulty of describing parallel algorithms. Based on Gemma and OpenCL, April can automatically execute Gemma described tasks with OpenCL.

This paper is organized as follows. Section II provides our motivations and previous works. In Section III, we present a novel parallel computation model based on general matrix operations - Gemma. In Section IV, we discuss the implementation and optimization of April, our parallel computing framework based on Gemma and OpenCL. Several application examples with test results and analysis are provided in Section V. Section VI concludes our work and makes the future research plans.

II. MOTIVATION AND RELATED WORKS

A. OpenCL: Open Computing Language

OpenCL is a programming language widely supported since 2009 [1]. It is promoted to unify the GPGPU programming process of AMD and NVIDIA, the programming of multi-core CPU and customized accelerators. That is to say, programs written following the OpenCL specifications can be executed on most mainstream multi-core CPU and GPGPU.

OpenCL is comprised of compilation chain and run-time library. OpenCL compilation chain can compile OpenCL programs statically or dynamically. The run-time library administers the load and execution of programs, initiates the data transmission and so on.

The programming model of OpenCL is intrinsically related to the architecture of GPGPU. For instance, the Work Item in OpenCL corresponds to thread in ATI Stream programming model, the Work Group to thread group and the Local Memory to LDS [2], [3].

B. Motivation for General-Purpose GPU Computing Architecture

Although GPGPU is powerful in computing and there is a well-build programming environment, but it is still difficult to write effective GPGPU accelerating application programs. Firstly, in order to parallelize their algorithms and map the parallel algorithms to the GPGPU threads, programmers must be familiar with the hardware architecture. Besides, the design of the synchronization of GPU threads and data transmission directly influences the effectiveness of the program. In addition, it is complicated to call runtime library which is designed for high generality. Figure 1 shows the programming flow based on OpenCL.

For the reasons above, a general computing architecture is necessary. A general computing architecture can be divided into three layers. The first layer is the Programming Model, which reduces the difficulty of designing parallel algorithm that needs to be mapped to GPGPU directly. The second layer is the Compiler, which compiles the program with the right Programming Model into the next layer of the framework. The last layer is the Runtime Library, which executes the program and schedules instructions such as data transmission.

An excellent architecture could significantly simplify programmers work. Fig.2 shows the workflow to develop a GPGPU program using an architecture under ideal circumstances.

In our novel architecture, Gemma represents the general parallel Programming Model and April consists the next two layers - Compiler and Runtime Library. We design the GPU computing architecture to simplify the programming process of effective high performance GPU programs.

C. Previous work on General-Purpose GPU Architectures

There is a lot of previous work about building up general purpose architectures of different designing goals. For example, MapReduce [4] is a well-known programming model, in which algorithms can be simply represented in two stages, *Map* and *Reduce*, both suitable for parallel systems. Programmers only need to map their algorithms to the model and define the two operations, Map and Reduce. Then MapReduce framework schedules the whole parallel system to complete the algorithm. Drayd [5] is a similar computing model, which generalize the two limited Map and Reduce stages in MapReduce

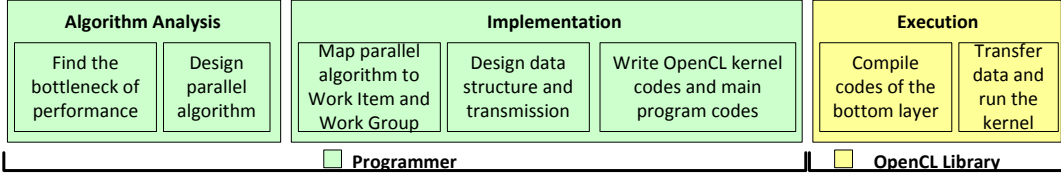


Fig. 1: OpenCL Programming Flow

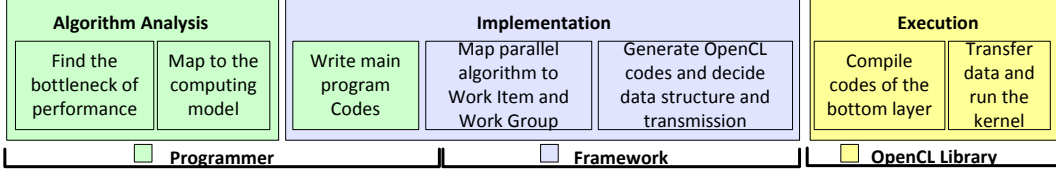


Fig. 2: Program Design Flow using ideal architecture based on OpenCL.

to multiple stages. Drayd uses DAG(Directed Acyclic Graph) to represent operations, in which edges represent transmissions of data.

There are other general computing frameworks for GPU, such as [6]–[8]. Mars [7] has a high compatibility with MapReduce of computer cluster, but targeting at variable-length strings, this framework is not effective with general computing. Catanzaro’s architecture [6] is a high performance GPU implementation of MapReduce, but the formats and regularity of data are strictly demanded, thus impair the generality. Tarditi’s architecture [8] is based on GPU Pixel Shader, which works effectively but its generality is restricted by the limitation of Pixel Shader.

III. GEMMA: MATRIX MULTIPLICATION LIKE PARALLEL PROGRAMMING MODEL

Gemma is a parallel Programming Model based on generalized matrix multiplication. In this section, we first introduce the basic idea and definition of the model, and then use several examples to illustrate how general algorithms get mapped to the model.

A. Generalization of Matrix Multiplication

Based on the common operations, we can define general matrix multiplication. If two matrices $A_{(U)m \times n}$ and $B_{(V)n \times p}$ and two mappings $\otimes : (U, V) \rightarrow S$ and $\oplus : (S, S) \rightarrow S$ follow the rules:

- 1) $\langle S, \oplus \rangle$ is a monoid, and the identity element is O_S
- 2) We can always find $O_U \in U, O_V \in V$, which makes $\forall v \in V, O_U \otimes v = O_S \forall u \in U, u \otimes O_V = O_S$

then we define matrix $C_{(S)m \times p}$

$$C_{(S)m \times p} : c_{u,v} = \bigoplus_{i=1}^n (A_{u,i} \otimes B_{i,v}) \quad (1)$$

as product of matrices A and B using \otimes and \oplus , denoted by $C = (A \times B)_{\otimes, \oplus}$.

In the definition above, O_U, O_V are called zero elements in U and V . According to the definition, if $U = V = S = \mathbb{R}$ and \otimes, \oplus are respectively multiplication and addition of real numbers, general matrix multiplication is same with matrix multiplication of real numbers with $O_U = O_V = O_S = 0$.

If we define U, V, S and \otimes, \oplus following certain rules, the general matrix multiplication above can solve a large number of computing problems. In programming, sets U, V, S can be data structures and \otimes, \oplus are binary functions. Especially, \oplus should follow the Combination Law, because $\langle S, \oplus \rangle$ is a monoid.

Several examples are given in the next subsection to show how to do parallel computing using general matrix multiplication.

B. Application Examples

The first example of generalized matrix multiplication is seeking the maximum element. Given n elements $a_i, (i = 1, 2, \dots, n)$, $a_i \in \mathbb{R}, (i = 1, 2, \dots, n)$, we define function $M(x, y)$ as follow:

$$M(x, y) = \begin{cases} x, & \text{if } x > y \\ y, & \text{otherwise} \end{cases} \quad (2)$$

Define matrix $A_{(\mathbb{R})1 \times n} : a_{1,j} = a_j$ and matrix $B_{(\mathbb{R})n \times 1} : b_{i,1} = 1$. The operation " \times " is multiplication of real numbers. Then $c = C_{(\mathbb{R})1 \times 1} = (A \times B)_{\times, M}$ is the maximum one of the n elements.

The next example is comparing and swapping. Take a pair of numbers (x_1, x_2) for example. To sort the numbers in a small-to-large order, we construct matrix $C_{\{0,1\}2 \times 2}$ as follows:

$$c_{1,1} = c_{2,2} = \begin{cases} 1, & \text{if } x_1 < x_2 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$c_{1,2} = c_{2,1} = \begin{cases} 1, & \text{if } x_1 \geq x_2 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Then the product matrix $Y = XC$ is the result of sorting.

C. Further Generalization of Matrix Multiplication

If all n -dimension function with range of S form a set, denoted by $F_{n,S}$, then matrix defined on $F_{n,S}$ is called n -dimension **Function Matrix**. If values of n variables form the set P , then function matrix $A_{(F_{n,S})}$ is the same with a common matrix, denoted by $A_{(S)}(P)$ or $A(P)$, which is a matrix defined on S . If two function matrices $A_{(F_{2,U})m \times n}$ and $B_{(F_{n,V})n \times p}$ and two mappings $\otimes : (U, V) \rightarrow S$ and $\oplus : (S, S) \rightarrow S$ follow the rules:

- 1) $\langle S, \oplus \rangle$ is a monoid, and the identity element is O_S
- 2) We can always find $O_U \in U, O_V \in V$, which makes $\forall v \in V, O_U \otimes v = O_S \forall u \in U, u \otimes O_V = O_S$

Then we define matrix $C_{(S)m \times p}$ as the product of function matrices A and B using \otimes and \oplus , denoted by $C = (A \times B)_{\otimes, \oplus}$.

$$C_{(S)m \times p} : c_{u,v} = \bigoplus_{i=1}^n (A(u, v)_{u,i} \otimes B(u, v)_{i,v}) \quad (5)$$

In comparison with the definition in Section III-A, here we generalize the common multiplier matrices to function matrices. When calculating value of the element at position u, v of output matrix C , matrices involved in calculating are the value of function matrices

at the position u, v . In other words, matrices involved in operation differ according to different objectives calculated.

Below is an example for the application of the multiplication of Function Matrices.

Suppose we have a matrix of real numbers $D_{(\mathbb{R})2 \times n}$. To seek the maximum number in row 1 and the minimum number in row 2 simultaneously, define function column vector $A_{n \times 1}(u, v)$ as follows:

$$a_{i,1}(u, 1) = \begin{cases} 1, & \text{if } u = 1 \\ -1, & \text{if } u = 2 \end{cases} \quad (6)$$

The values of j and v are always 1. Using the function $M(x, y)$ defined in III-B, we can get the result immediately: $(c_1, c_2)^T = C = (D \times A)_{\times, M}$. The maximum number of row 1 is c_1 and the minimum of row 2 c_2 .

IV. APRIL: GEMMA AND OPENCL BASED PARALLEL COMPUTING PROGRAMMING FRAMEWORK

April is a computing framework based on the model of Gemma and the framework of OpenCL. As discussed in Section II-B, April consists two layers of a general computing framework. The first one is the Compiler, which generates OpenCL codes from the representations of Gemma-based matrix operations (MOPs). The second layer is the Runtime Library, which control the data flow and execution the OpenCL program compiled from the generated codes. April also includes the optimization schemes corresponding to the two layers. In the Compiler layer, April simplifies the programming codes by letting user describing the application into MOPs by Gemma. In the Runtime Library layer, April optimizes the arrangement of each MOP by traversing a Directed Acyclic Graph (DAG) constructed by MOPs. April also selects the best matrix representations for each MOP by comparing their performance models.

Plan is the unit of April programs, which is formed by matrix operations organized by a linear fashion. An operation can be matrix construction (definition), rearranging multiplication and output. The linearity of Plan excludes the branch and loop structures, which need to be designed by users. The workflow of April is showed in Fig.3. The first procedure, Operations Defining, is controlled by users. The rest three procedures, Decision Making, Kernel Codes Producing and Program Executing, can be automatically completed by April.

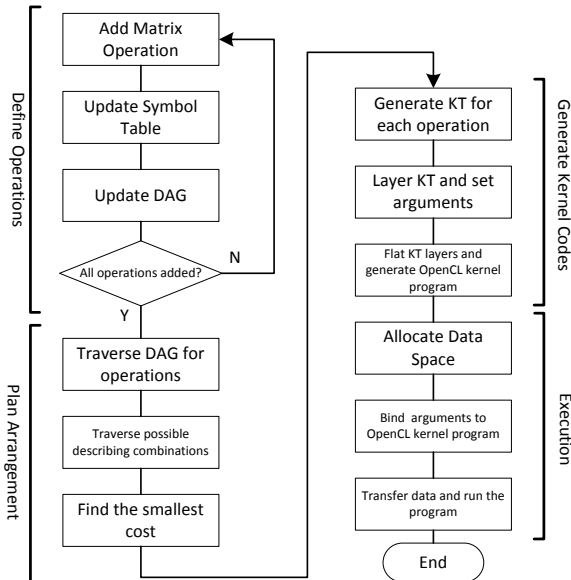


Fig. 3: April Workflow

In the beginning of this section, we begin with a brief introduction of the matrix representation methods in April in Section IV-A. We then presents the details of the 4 procedures of April's workflow in Section IV-B, IV-C, IV-D and IV-E. Finally, in Section V-C we presents the performance model of April and the optimization schemes.

A. Matrix Representation

The key to April is effective methods of matrix storing. The matrix representation methods are the foundation of April's Runtime-level optimization. In this section, we give four methods to store a matrix.

BUF: The BUF method is for dense matrix storage. In BUF mode, a matrix is stored in a continuous space in the memory. Matrices stored as BUF can be easily accessed but demand a larger storage space. Function matrices cannot be represented as BUF.

XY: In XY mode, we use a function with two arguments x, y ¹ to describe a matrix. Stored with the method of XY, matrices can be accessed randomly, and it can represent function matrices. The storage space depends on the function describing the matrix. If the matrix is special and regular, such as diagonal matrix and triangular matrix, then representing it in XY will require much smaller space.

To describe a function matrix using method XY, we need to make a function `getXY` with input arguments x, y, u, v , which returns the value of element(function) at position x, y of matrix (at point u, v).

CSR: We usually use the CSR method to store sparse matrix [9], which only stores non-zero element in the order of row priority. We need three function to represent a matrix using CSR:

- 1) `rowsize(y)`, returning the number of non-zero elements in row $y + 1$.
- 2) `col(y, idx)`, returning the column index of the $(idx + 1)$ th non-zero element in row $y + 1$.
- 3) `val(y, idx)`, returning the value of the $(idx + 1)$ th non-zero element in row $y + 1$.

y, idx and return value of function `col` all begin with 0.

We can only obtain the value and position of non-zero element of matrix stored with CSR. CSR is better at representing special matrices with higher sparsity. As zero element can be skipped, the speed of sparse matrix multiplication can be greatly improved.

CSC: Similarly with CSR, CSC is a storing method for sparse matrix in the order of column priority.

CSR and CSC are different in the order of storing and accessing non-zero element and thus making CSR matrix better for premultiplication and CSC matrix for postmultiplication.

B. Plan Definition

A MOP is constructed by users as an independent object. There are three kinds of MOPs. The first one is the Construction Operation, which has one output matrix and no input matrix. All kinds of data can be bound to the matrices constructed. The second one is the Computing Operation, which has one output matrix and one or more input matrices. The typical example is matrix multiplication with two input matrices and one output matrix. The last one is the Output Operation which has one input matrix and no output matrices. The typical example is writing a BUF matrix to memory or getting the OpenCL Buffer Handle directly.

In all three operations, users can define data structure and interface function with one or more matrix representation methods, such as BUF, XY, CSR and CSC. For general operations, April can support all the possible representation methods. From the supported and user-defined representation methods, April will choose the best one to minimize operation costs.

¹In this article, x is for column and y is for row beginning with 0

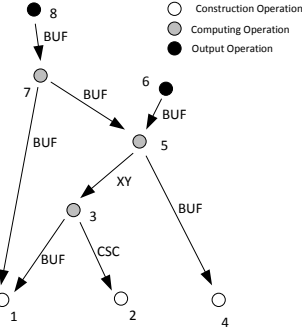


Fig. 4: DAG of Matrix Operations

When a MOP is added to a Plan, a vertex represents that MOP is added to a DAG. In the DAG of operations, every vertices represent MOPs and each directed edge represents the relationship between MOPs or their data flow. For example, an edge from V_1 to V_2 represents that the input matrix of operation V_1 is the output of the operation V_2 . The out-degree of each vertex equals the number of input matrices of the operation and the in-degree the number of outputs. Each edge of the DAG indicates the representation method of the destination vertex's output matrix. Different representation methods can result in different performance, which is discussed in Section V-C. Figure 4 shows an example of a DAG which depicts the construction and computing operations of three matrices, as well as two output operations. By each edge in the graph is the representation method of matrix.

C. Arrangement of MOPs

After the addition of operations, April begins to arrange the execution order and IO matrix representation methods of these operations.

We rearrange the order of operations to get rid of unnecessary operations. The output operation is focused first and then begins the breadth-first post-order traversal to arrange the operations in DAG that depend on the output operation in order. If there are more than one output operations, then start the traversal from them and skip the nodes accessed. In the meantime, the last position accessed of every node is recorded, so that the memory space can be freed when the output data are no longer needed.

As showed in Figure 4, the breadth-first Postorder Traversal begins from node 6 and we get a sequence:

$$1, 2, 3, 4, 5, 6$$

Then traverse from node 8 and we get:

$$7, 8$$

After linking, we get the final operation order:

$$1, 2, 3(2), 4, 5(3, 4), 6, 7(1, 5), 8(7)$$

The nodes in the brackets are no longer needed(*timeoutNodes*).

D. Kernel Codes Generation

With operations in Plan and the matrix representation methods settled, the kernel codes can be generated. An independent kernel program is based on a series of operations with an output matrix represented by BUF. Output matrices represented by the other three methods can only be used as input matrices by the following operation, but they cannot be the output matrices of a kernel program.

To describe the operations of layered structure in Plan, April uses Kernel Template(KT), which is a group of codes containing

template variables that can be replaced. We can generate the real codes by replacing the template variables. The template variables can be replace by other KT, thus forming the layered template structure.

E. Plan Execution

A Plan can be executed after all the kernel codes of matrix operations in the Plan are generated. The execution of a Plan includes data transferring, I/O arguments binding and program executing.

Each operation is executed in the order that rearranged as it is described in Section IV-C. There are three procedures in the Plan Execution, `prepare`, `execute` and `cleanup`, which respectively allocate the output space, execute the operation and free the output space. The allocation and deallocation of input space are accomplished by the parent node.

F. Optimizations

We begin by the performance model of a April Plan. Different matrix representations have the different running cost and output cost. The cost of a node is estimated according to the number of elements accessed in memory when outputting an element, and calculated based on the output method and the output costs of the child nodes.

Assume each operation has a running cost c_r and an output cost c_o , and all the matrices are $n \times n$ square. When the output mode of node p is BUF, the output matrix is calculated by multiplying two dense matrices, and the output matrix is write to the memory instantly. So the output cost $c_o(p) = 1$ and the running cost is

$$c_r(p) = n^3 * \sum_{\text{for each child node } i \text{ of } p} c_o(i) \quad (7)$$

When the output mode of node p is XY, the output matrix is only a function of the input matrices, so the running cost $c_r(p) = 0$ and the output cost is

$$c_o(p) = n * \sum_{\text{for each child node } i \text{ of } p} c_o(i) \quad (8)$$

The performance model of CSR and CSC mode is estimated by the definition of `rowsize`, `col` or `row` and `val` in the same way with BUF and XY. The total Plan running cost is the sum of running costs of all the nodes.

Take the Plans in Fig.5 for an example. Assume that operation 3,5,7 are all matrix multiplications, the computing time of \otimes, \oplus can be ignored, matrices are all square matrix of order n and the sparse matrix constructed by operation 2 is a diagonal matrix.

For plan (a), the output costs of construction operation 1,2,4 are:

$$c_o(1) = c_o(2) = c_o(4) = 1 \quad (9)$$

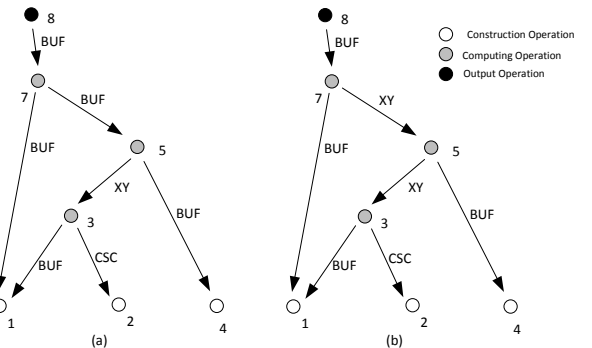


Fig. 5: Different costs of different I/O matrix representation methods

Because the data need to be transmitted, assume that:

$$c_r(1) = c_r(4) = n^2, c_r(2) = n \quad (10)$$

The output cost of operation node 3(XY) is:

$$c_o(3) = c_o(1) + c_o(2) = 2 \quad (11)$$

The node 5 output a BUF matrix with the output cost $c_o(5) = 1$ and the running cost:

$$c_r(5) = n^3 * (c_o(3) + c_o(4)) = 3n^3 \quad (12)$$

The node 7 output a BUF matrix with the output cost $c_o(7) = 1$ and the running cost:

$$c_r(7) = n^3 * (c_o(1) + c_o(5)) = 2n^3 \quad (13)$$

The node 8 transmit the result with the running cost $c_r(8) = n^2$ The total cost of the Plan (a) is:

$$c_{r(a)} = \sum_{\text{for each node } n} c_r(n) = 5n^3 + 3n^2 + n \quad (14)$$

For Plan (b), the only difference is the output mode of node 5. Therefore, the running cost of node 5 is 0 and the output cost is:

$$c_o(5) = n * (c_o(3) + c_o(4)) = 3n \quad (15)$$

The running cost of node 7 is:

$$c_r(7) = n^3 * (c_o(1) + c_o(5)) = n^3 + 3n^4 \quad (16)$$

The total cost of the Plan is:

$$c_{r(b)} = \sum_{\text{for each node } n} c_r(n) = 3n^4 + n^3 + 3n^2 + n \quad (17)$$

Compared with (a), the cost of (b) is bigger, so (a) is a better Plan.

April traverse all the possible representation methods of I/O matrices, estimate the cost of each combination and choose the most effective one.

V. EXPERIMENTAL RESULTS

In this section, we presents some of our experimental results to discuss the advantages of our generalized computing architecture: Gemma and April.

A. Experiment Platform and Test Methods

The following experiments are conducted by the same test computer, equipped with AMD PhenomII 965 64-bit 3.4GHz quad-core processor, 8GB DDR3 1333MHz RAM and ATI RadeonHD 5870. The interface between the graphics card and processor is PCI Express x8 with bandwidth of 2.5GB/s. The operating system kernel is Linux 2.6.31 and we use GCC 4.4 as compiler. The driver for GPU is Catalyst 10.5 and the version of OpenCL SDK is ATI Stream SDK 2.1.

All testing programs are executed repeatedly and we calculate the average performance. The executing time of the Plan, including time of data transmission and API, reflects the computing performance of April, but the compiling time is excluded.

B. Improvement of Code Amount

In the section, we discuss code amount differences between pure OpenCL and April. The application is a simple 1024×1024 matrix multiplication program.

$$R = AB \quad (18)$$

A and B are 1024×1024 matrix with single-precision floating-point elements.

TABLE I: 1024 × 1024 Float Matrix Multiplication Code Amount

Platform	Scheduling Codes	Kernel Codes(Computing Codes)
C	10	10
April	40	34(auto-generated)
OpenCL	1050	96

TABLE II: Comparison I of Different Output Describing Methods

Plan	Time(ms)	Performance(GFLOPs)	Cost
(a)	65.8	32.637	2152726528
(b)	90.3	23.782	4297064448
(c)	163.8	13.110	5370806272

Table I shows the code amount of matrix multiplication on different platforms. Compared with pure OpenCL, April significantly reduces the amount of code, because April can automatically finish much work, such as initialization, setting and scheduling. In this case, matrix multiplication is the basic operation of April. Users do not need to write any kernel codes by themselves. In fact, even if it is necessary for users to write some kernel codes, the difficulty of programming can be reduced by April.

C. Comparison of Different I/O Methods

As discussed in Section , April can choose for every operation the best output methods among BUF, XY, CSR and CSC. In this section, we discuss the influence of different output methods on computing performance.

Take the computing task of matrix multiplication for example.

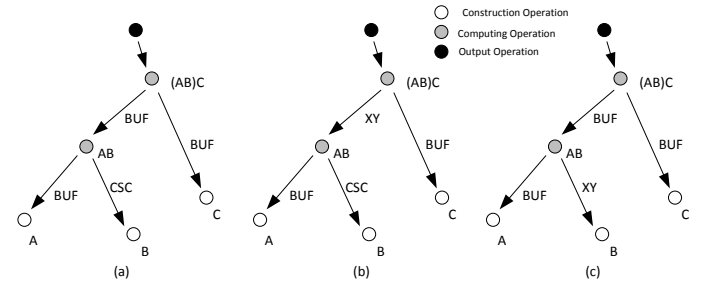


Fig. 6: DAG of Different Output Methods

We use DAG to represent the computing task, as showed in Figure 6. There are three different Plans among which Plan (a) is chosen by April. Table II shows the performance and cost estimated by April of three different Plans. We find a big difference among different Plans and April chooses the best one.

Take the computing task showed in Figure 5 for another example:

$$R = M_1 ((M_1 M_2) M_4) \quad (19)$$

All matrices are 512×512 . Matrix M_2 is a diagonal matrix with constant diagonal elements, so $c_o(M_2) = 0$. The cost of the computation is $2(2n^3 - n^2) + n^2 = (4n^3 - n^2)$ FLOPs.

Table III shows the result of this experiment. We can see the performance of different Plans are different and April chooses the best one.

TABLE III: Another Comparison of Different Output Methods

Plan	Time(ms)	Performance(GFLOPs)	Cost
(a)	20.6	26.049	537395200
(b)	3094	0.173	137573695488

D. Full Example: Bitonic Sort

Bitonic Sort [10] is a sorting algorithm of high parallelism. For a group of data $v_i, i = (0, 2, \dots, 2^b - 1)$ with the length $N = 2^b$, Bitonic Sort can finish the sorting by b steps. Each step s includes s parallel sorting procedures and each procedure includes $N/2$ comparison and swapping. As a result, the complexity of the algorithm is $O(N(\log N)^2)$.

The algorithm of Bitonic Sort can be mapped to the Programming Model of Gemma. Assume $N = 2$, we can finish the sorting in the order from small to large:

$$\begin{pmatrix} r_0 & r_1 \end{pmatrix} = \begin{pmatrix} v_0 & v_1 \end{pmatrix} X^{(1,1)} \quad (20)$$

The matrix $X^{(1,1)}$ is defined as follow:

$$X^{(1,1)} = \begin{cases} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \text{if } v_0 < v_1 \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & \text{otherwise} \end{cases} \quad (21)$$

The best performance is achieved if this swapping matrix is described with CSC. The method CSC needs three functions defined, `colsize`, `row` and `val`. Obviously, there is only one element with value 1 in each column of a swapping matrix. Algorithm 1 shows how to define the three functions of the swapping matrix of Bitonic Sort $X^{(s,p)}$. s and p are current step and procedure, $s = 1, 2, \dots, b; p = 1, 2, \dots, s$, as showed in Algorithm 1.

Algorithm 1: Swapping Matrix of Bitonic Sort with CSC: Step s and Procedure p

```

function colsize(x)
  | Return 1;
end
function row(x, idx)
  |  $x' \leftarrow x \text{ XOR } 2^{s-p}$ ;
  |  $c_1 \leftarrow (v_x < v_{x'})$ ;
  |  $c_2 \leftarrow (x \text{ AND } 2^{s-p} == 0)$ ;
  |  $d \leftarrow (x \text{ AND } 2^s == 0)$ ;
  | if  $c_1 \text{ XOR } c_2 \text{ XOR } d$  then Return  $x$ ;
  | else Return  $x'$ ;
end
function val(x, idx)
  | Return 1;
end

```

When programing Bitonic Sort using April, we input the original data by row vector and each procedure is a matrix described with CSC. For each bitonic sort p in each step s , we initialize the sort matrix $X^{(s,p)}$ and add the MOP to the Plan. All the MOPs are added to Plan by loop structure.

Figure 7 shows the performance comparison of CPU sorting and GPU Bitonic Sort based on April, using the logarithm of data amount as abscissa and the data are all of single-precision floating-point. We use the function `sort` in STL as the CPU sorting method. We can see from the figure that GPU sorting works no better than CPU when the amount of data are small, mainly because 1)it takes time to transmit data between main RAM and VRAM and 2)there are several procedures in Bitonic Sort, each a GPU kernel program, so the cost of calling API cannot be ignored. In general, the GPU sorting has a high speedup, when the data are large in scale. Moreover, April's performance is in the same order of magnitude with other optimized GPU sorting implementations [11].

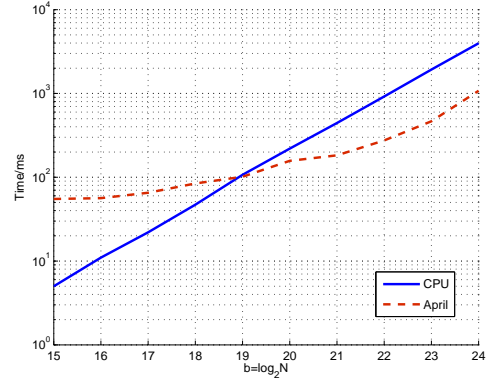


Fig. 7: Performance comparison between the CPU and April implementation of Bitonic Sort. The dashed line shows the results of April version of Bitonic Sort, which is slower than CPU version when the problem scale is small but faster when the scale becomes large.

VI. CONCLUSION AND DISCUSSIONS

In this article, we propose a novel parallel computing architecture. The architecture includes Gemma, a general parallel Programming Model, and April, a computing framework based on Gemma and OpenCL. Gemma uses matrix operation, especially matrix multiplication, to describe general computing tasks. Because the Gemma model is based on matrix, the parallelism is unrelated with the hardware platform. In other words, computing tasks described by Gemma can be applied to any platform including multi-core CPUs or GPGPUs. April can choose the proper combination of output methods of matrices, so that the cost of the whole plan is optimized.

In the future, we have the following research objectives. Firstly, we will explore the possibility of implementing the computing tasks on multiple devices such as multiple CPUs and GPUs. Secondly, the computation can be automatically divided into several parts and controlled by scheduling schemes, especially when the data scale exceeds the Video RAM (VRAM) capacity. Thirdly, we can extend the computation from single to multi-machine platforms and achieve computation speedup by utilizing computer clusters.

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